Tour of the Typelevel

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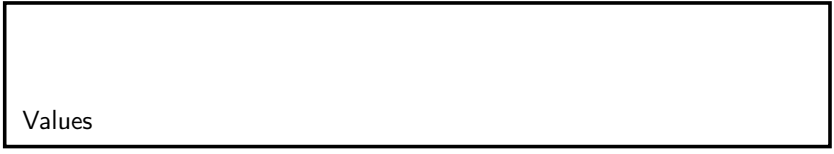
Tour of the Typelevel

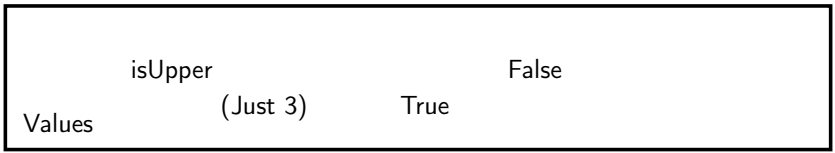
Tonight:

- Several common type system extensions.
- Motivating examples.
- Dropping signposts.
- Less scary when you see them in libraries.
- Please keep arms and legs inside the vehicle at all times.

Values, Types, and Kinds



Values, Types, and Kinds



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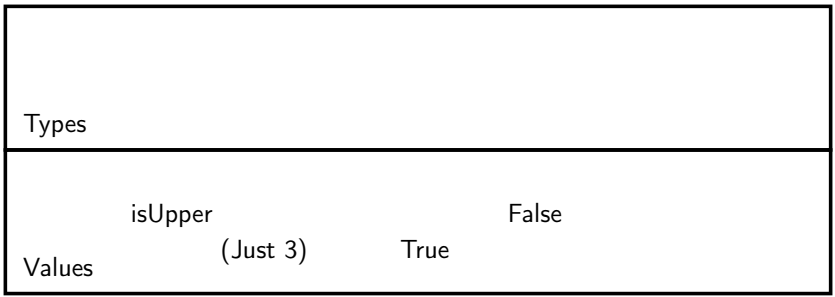

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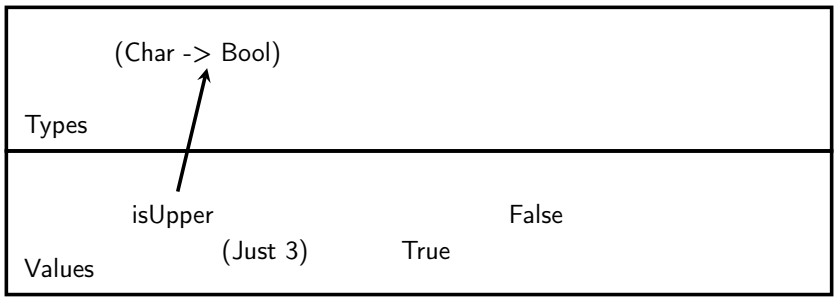

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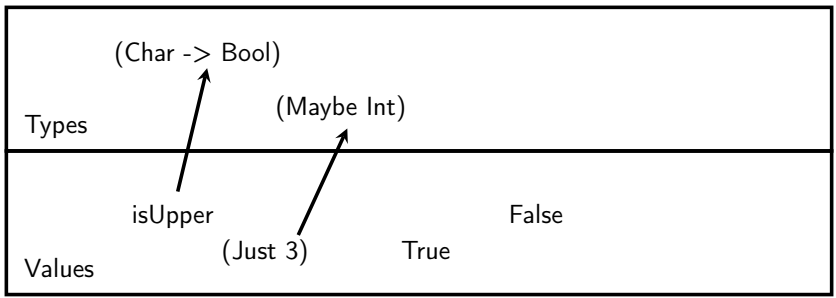

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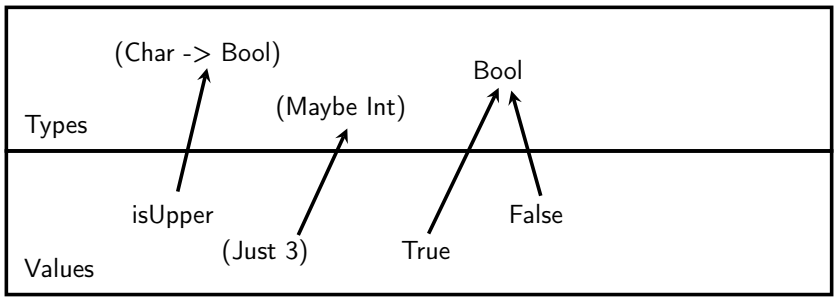

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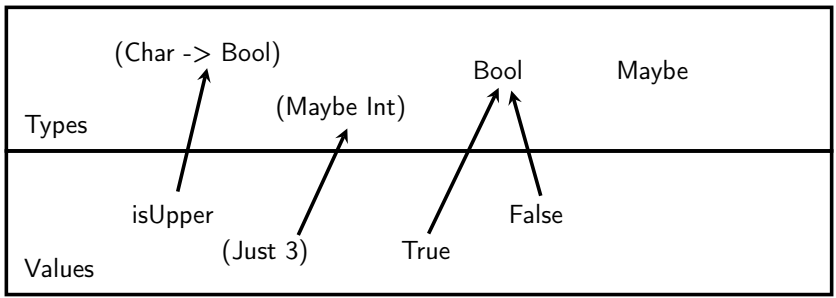

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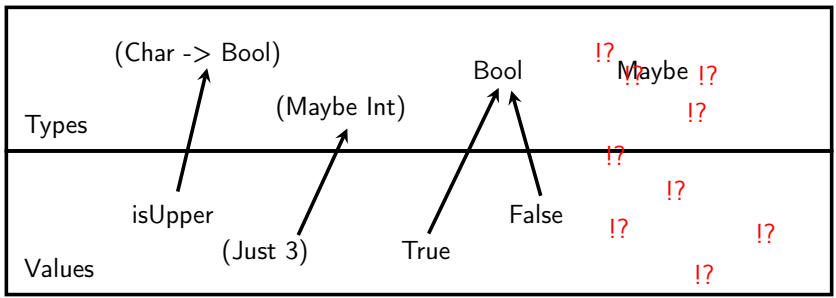

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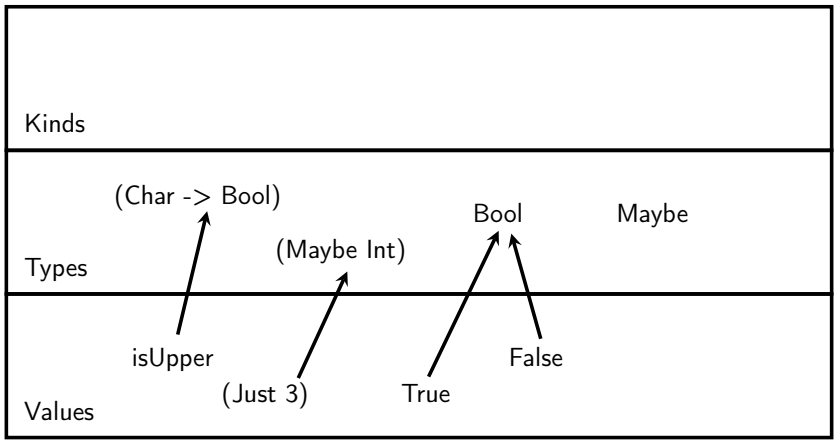

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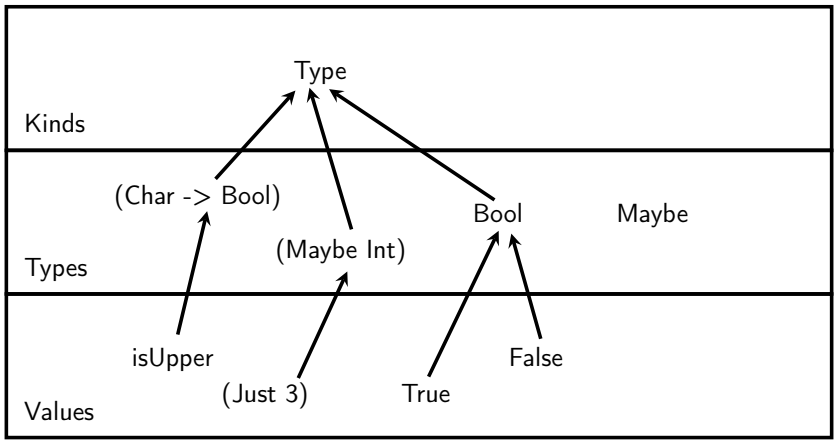

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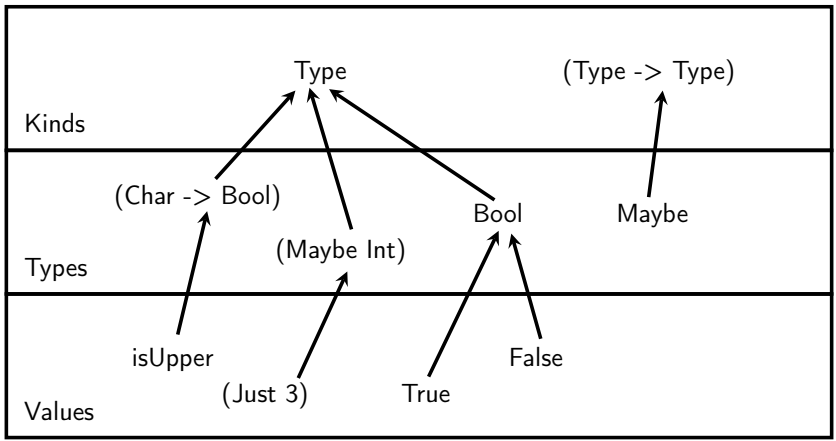


Values, Types, and Kinds

What did we learn?

- *Types* classify *values*.
- *Kinds* classify *types*.
- Types of kind `Data.Kind.Type` may have values.
  - Also called `*`, but this will (eventually) go away.
  - *May*, not *must*: `Void` has no values.
- Ask GHCi for the kind of a type:

```
> :kind Maybe
Maybe :: * -> *
```

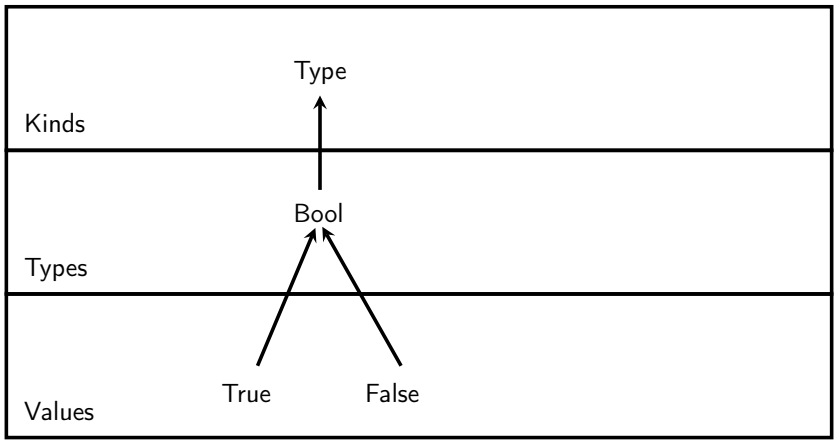
Kind Signatures

- Values may have *type signatures*.
- Types may have *kind signatures*:

```
{-# LANGAUGE GADTSyntax, KindSignatures #-}

data Either (a :: Type) (b :: Type) :: Type where
  Left :: a -> Either a b
  Right :: b -> Either a b
```

DataKinds



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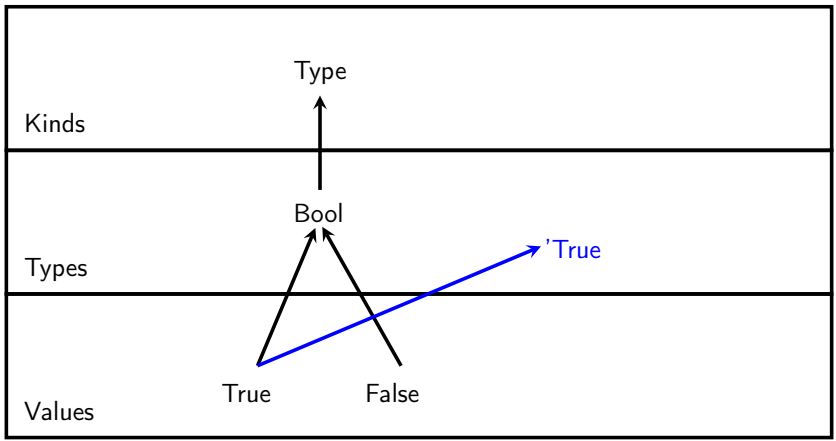

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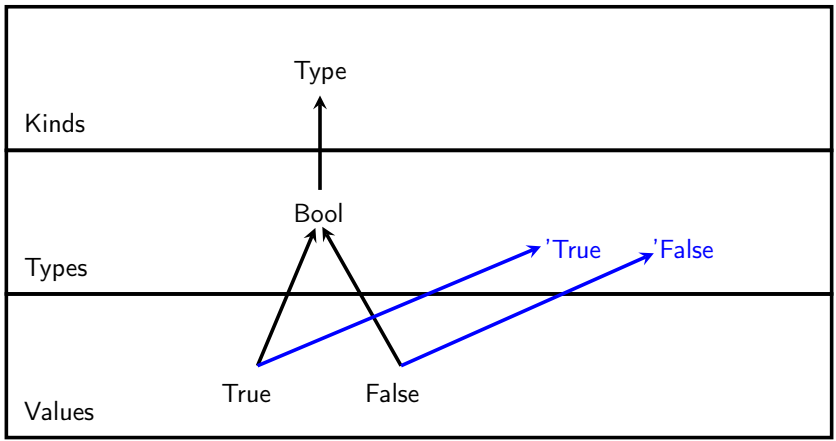

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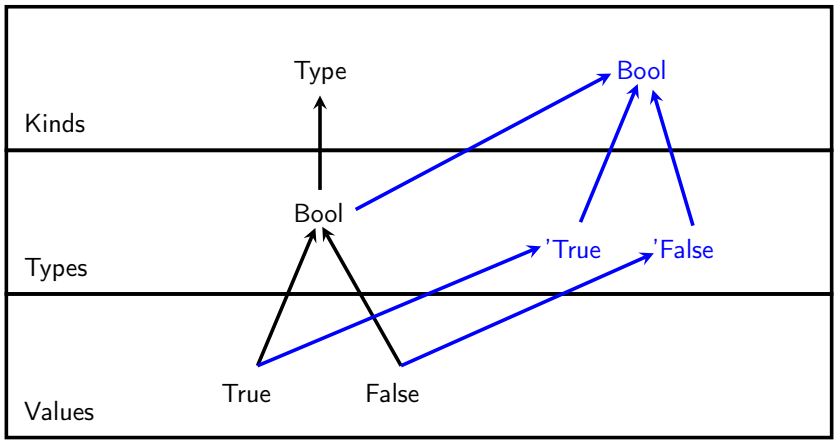

DataKinds

- What does `data Either a b = Left a | Right b` do?
- It introduces:
  - The type constructor `Either :: Type -> Type -> Type`
  - Two data constructors:
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- If we set `{-# LANGUAGE DataKinds #-}`, we also get:
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- `DataKinds` also gives you type-level strings (kind `Symbol`) and naturals (kind `Nat`).

DataKinds Example

Suppose:

- I have a data structure, stored on disk.
- It is *really important* that I verify it before use.
  - e.g. Moxie's Cryptographic Doom Principle
    `https://moxie.org/blog/the-cryptographic-doom-principle/`
- How do I enforce this?

DataKinds Example: Unsafe API

```
data Structure = -- ...

loadStructure :: FilePath -> IO Structure
loadStructure = -- ...

verify :: Structure -> Bool
verify = -- ...
```

DataKinds Example: Verified Reads

```haskell
{-# LANGUAGE DataKinds #-}

data Verification = Unverified | Verified
data Structure (v :: Verification) = -- ...
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loadStructure = -- ...

verify
  :: Structure v
  -> Maybe (Structure 'Verified)
verify = -- ...
```

DataKinds Example: HList

```
{-# LANGUAGE DataKinds #-}
{-# LANGUAGE GADTs #-}
{-# LANGUAGE KindSignatures #-}
{-# LANGUAGE TypeOperators #-}

data HList (xs :: [Type]) :: Type where
  HNil :: HList '[]
  HCons :: x -> HList xs -> HList (x ': xs)
```

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```

```
> :t HCons True (HCons "Hello" HNil)
HCons True (HCons "Hello" HNil) :: HList '[Bool, [Char]]
```

ConstraintKinds

`{-# LANGUAGE ConstraintKinds #-}` lets us use the kind `Constraint` in contexts:

- It gives us a new kind called `Constraint`:

```
> :k Show
Show :: * -> Constraint
> :k Monad
Monad :: (* -> *) -> Constraint
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ConstraintKinds

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- Tuples of `Constraint` are also of kind `Constraint`:

```
> :k (Eq Int, Show Int)
(Eq Int, Show Int) :: Constraint
```

ConstraintKinds

- We can write constraint aliases using `type`:

```
type Stringy a = (Read a, Show a)
roundtrip :: Stringy a => a -> a
roundtrip = read . show
```

- Ed Kmett has a library `constraints` for playing with these, and a talk "Type Classes vs. the World" (skip to 46:00).

Multi-Parameter Type Classes

- What is a type class?
  - It's a property on types:
    - `Int` is in `Num`, `Show`, ...
  - It may have methods for ad-hoc polymorphism:
    - Implementation chosen by the argument type
- Why restrict ourselves to one type?

Multi-Parameter Type Classes

- Set `{-# LANGUAGE MultiParamTypeClasses #-}`.
- Type classes are now *N*-ary relations on types:

```
class Resource idType resourceType where
  url :: idType -> String
  get :: idType -> IO resourceType

newtype PostId = -- ...
data Post = -- ...
instance Resource PostId Post where
  -- ...
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```

- What is the type of `get (pid :: PostId)`?
  - We want `IO Post`.
  - But type classes are an open world!
  - How can GHC know there's no other instances?

Functional Dependencies

- Set `{-# LANGUAGE FunctionalDependencies #-}`.
- We can now say which parameters force which other parameters:

```
class Resource i r | i -> r, r -> i where
  url :: i -> String
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- What is the type of `get (pid :: PostId)`?
  - `IO Post`.
- What is the type of `pid` in `get pid :: IO Post`?
  - `PostId`.

Type Families

- Type families are type-level functions.
- Think of them as type aliases that can inspect their argument.
- Set `{-# LANGUAGE TypeFamilies #-}`

Type Families: Closed Type Families

- The simplest:

```
type family
  Append (xs :: [k]) (ys :: [k]) :: [k]
  where

  Append '[] ys = ys
  Append (x ': xs) ys = x ': (Append xs ys)
```

Type Families: Closed Type Families

- The simplest:

```
type family
  Append (xs :: [k]) (ys :: [k]) :: [k]
  where

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```

- `:kind!` will force GHCi to expand a type-level expression:

```
> :kind! Append '[Int] '[Bool]
Append '[Int] '[Bool] :: [*]
= '[Int, Bool]
```

Type Families: Open Type Families

- Instead of providing all the equations, allow anyone to add their own.

```
{-# LANGUAGE DataKinds     #-}
{-# LANGUAGE PolyKinds     #-}
{-# LANGUAGE TypeFamilies #-}

data Permission = P1 | P2
type family
  Permissions (act :: k) :: [Permission]

-- In some other module...
data Action = A1 | A2
type instance Permissions 'A1 = ['P1, 'P2]
type instance Permissions 'A2 = '[]
```

Type Families: Associated Types

- Add type aliases to type classes.

```
class MonoFunctor m where
  type Elem m :: Type
  omap :: (Elem m -> Elem m) -> m -> m

instance MonoFunctor Text where
  type Elem Text = Char
  omap = -- ...
```

- Remark: This is a good motivating example for associated types, but `Traversals` (from `lens`) are a better solution to this problem.

Data Families

- Also turned on by `{-# LANGUAGE TypeFamilies #-}`.
- Write `data` instead of `type`: `data family ... where` etc.
- Let you create new data types based on type arguments.
- Used much less often.

Proxy

- Consider this function:

```
normalise :: String -> String
normalise = show . read
```

- What instances of `Read` and `Show` do we use?

Proxy

- Consider this function:

```
normalise :: String -> String
normalise = show . read
```

- What instances of `Read` and `Show` do we use?
- We can use a `Proxy` to explicitly choose an instance.

```
{-# LANGUAGE PolyKinds, KindSignatures #-}

data Proxy (a :: k) = Proxy
```

Proxy Example: String Normalisation

- Consider this version:

```
{-# LANGUAGE ScopedTypeVariables #-}

normalise
  :: forall a . (Read a, Show a)
  => Proxy a
  -> String
  -> String
normalise _ s = show (read s :: a)
```

Proxy Example: Value of a Symbol at Runtime

```haskell
{-# LANGUAGE DataKinds, TypeApplications #-}

import Data.Proxy
import GHC.TypeLits

hello :: String
hello = symbolVal (Proxy :: Proxy "hello")

hello' :: String
hello' = symbolVal (Proxy @"hello")
```

Summary

- `DataKinds` lifts `data` declarations into new types and kinds.
- `ConstraintKinds` lets you use types of kind `Constraint` inside contexts.
- `GADTs` let you fiddle the type variables in data constructors.
- `MultiParamTypeClasses` turn type classes into *relations* on types. Use `FunctionalDependencies` to help things along.
- `ScopedTypeVariables` lets you use type variables inside function bodies.
- `TypeFamilies` lets you write type-level functions.
- The `Proxy` type lets you pass additional type information to functions.